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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A RELIABILITY MODEL FOR STORED ITEMS
REQUIRING REWORK

by

James Guy Bohannan

March 1977

Thesis Advisor: Glenn F. Lindsay

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) A Reliability Model for Stored Items Requiring Rework.	5. TYPE OF REPORT & PERIOD COVERED (9) Master's Thesis,
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) James Guy/Bohannan	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL POSTGRADUATE SCHOOL Monterey, California 93940	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS NAVAL POSTGRADUATE SCHOOL Monterey, California 93940	12. REPORT DATE (11) March 1977
	13. NUMBER OF PAGES 50 (12) 51p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) NAVAL POSTGRADUATE SCHOOL Monterey, California 93940	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability Model

Inventory Management

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical model is developed for a system of stored items which are periodically reworked to improve their reliability. Expressions are developed for the expected reliability of the stored items and for the probability that the reliability of an item will exceed a required reliability. These expressions are developed first for the transient "start-up" phase where new items enter an initially empty system, then for the continuous rework phase where items are being reworked and returned to storage, and finally for the

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20. Abstract (Continued)

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transient replacement phase where the old items are being replaced by new or different items. Numerical examples are presented to demonstrate the use of the expected reliability and probability expressions.

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A RELIABILITY MODEL FOR STORED ITEMS

REQUIRING REWORK

by

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Lieutenant, United States Navy
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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

from the

NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

A mathematical model is developed for a system of stored items which are periodically reworked to improve their reliability. Expressions are developed for the expected reliability of the stored items and for the probability that the reliability of an item will exceed a required reliability. These expressions are developed first for the transient "start-up" phase where new items enter an initially empty system, then for the continuous rework phase where items are being reworked and returned to storage, and finally for the transient replacement phase where the old items are being replaced by new or different items. Numerical examples are presented to demonstrate the use of the expected reliability and probability expressions.

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I. INTRODUCTION

A mathematical model is developed in this thesis for a system of stored items which are periodically reworked to improve their reali-ability. In such a system the reliability of the stored items will depend upon the realiabilities of the items at the time of acquisition and the effectiveness of the rework.

The system considered in this thesis consists of a quantity of stored items together with a rework mechanism which acts to increase the reliability of items submitted to it. Such a system, as shown in Figure (1), might include a stock of ordnance which is acquired, stored, and periodically reworked but not expended except for war-time use.

In this thesis, reliability is considered as an attribute of an item which deteriorates over time. If an item is removed from storage and tested, it is presumed that if the reliability of the item is sufficiently high, the item will work, if not, it won't.

There are several ways to portray the effect of rework on an item. One way is for the rework mechanism to raise an item's reliability to a certain level which is independent of the item's reliability prior to rework. Raising reliability to a certain level is a plausible representation when components or parts are replaced rather than repaired, such as batteries or propellant crystals in rockets [1]. Another way, and the one developed in this thesis, is for the rework mechanism to achieve an increase in reliability which is proportional to both the item's reliability before rework and the effectiveness of the rework mechanism.

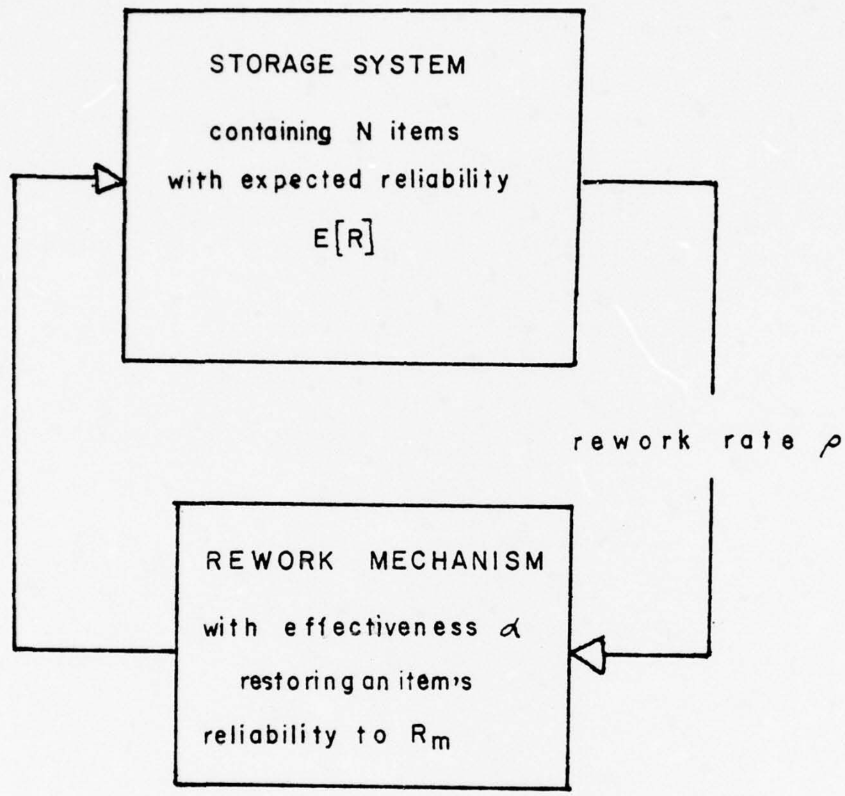

FIGURE 1. The Closed Storage-Rework System

This type of rework mechanism might exist where major assemblies or subassemblies are tested and repaired rather than replaced. Under these circumstances, it may be possible for such a rework mechanism to improve the item's reliability to where it is "better than new", as "good as new", or not as "good as new". In this last case the reliability will continue to deteriorate until the items need to be replaced rather than reworked.

An inventory manager, or combat planner in the case of stored ordnance, might have a need to determine the expected reliability of a quantity of stored items and the probability that an item selected at random will have reliability exceeding some reliability requirement. This information could be used in developing a replacement policy or in determining the need to either upgrade the effectiveness of the rework mechanism or increase (or decrease) the rate of rework to establish a certain level of expected reliability.

A general rework model is developed in Chapter II which relates the reliability of an item following rework to its initial reliability at acquisition, the number of times an item has been reworked and the "quality" of effectiveness of the rework process.

In the next three chapters expressions are developed for the expected reliability of the system and the probability that an item's reliability exceeds a given reliability requirement. The "start-up" phase is discussed in Chapter III. This is the special case of acquiring new items and placing them into the storage system, building up the inventory. The "steady-state" or continuous rework process where the storage system is filled and items are being removed, reworked and

returned to storage is addressed in Chapter IV. The replacement phase where the old items are being removed and replaced by new items or old items which have been reworked through an improved rework process is discussed in Chapter V. Numerical examples are presented at the end of each chapter to demonstrate the use of the expected reliability expressions and probability statements.

Conclusions and recommendations for further study are offered in Chapter VI.

II. DEVELOPMENT OF THE REWORK MODEL

In this Chapter the general rework model is developed which will relate the reliability of an item following a rework to its initial reliability upon entering the system, the number of times it has been reworked, and the quality or effectiveness of the rework process.

One way to represent the effect of the rework process is to have the process improve an item's reliability R by a percentage of its reliability prior to work. In other words, the process would reduce the unreliability (1-R) of the item by an amount α which may be considered a measure of the effectiveness of the rework process.

The reliability of an item just after it has been reworked R_s can than be expected as

$$R_s = R + \alpha(1-R) \ ,$$

or

$$R_s = \alpha + (1-\alpha)R \ , \qquad 0 \leq \alpha \leq 1 \ . \tag{1}$$

The interpretation of the rework effectiveness α is that the greater its value, the more effective the rework process.

One assumption made in this thesis is that all failures are random. This implies (1) that either there are no early failures or some form of "burn-in" is used to eliminate early age failure in stored items and (2) that either there are no wearout failures or the time to occurance of wearout is much longer than the projected unit operating time or time until the next scheduled rework. This seems to be a reasonable assumption for the system under consideration because the items are in storage,

not an operating environment, and removed from storage only for rework (or for expenditure in combat, if the items are ordnance). Because of this, the reliability function is then as exponential function [2].

Let $R(t)$ represent the reliability of an item of age t. Then, because of the random failure assumption the reliability function is

$$R(t) = e^{-(a+bt)} \quad , \quad t \geq 0 .$$

The initial reliability R_0 is

$$R_o = R(0) = e^{-a} \quad ,$$

and thus

$$R(t) = R_o e^{-bt} \quad , \quad b > 0 \quad , t \geq 0 . \qquad (2)$$

The parameter b will determine the magnitude of the loss in relability during a storage period of time t. The value of b depends on the nature of the item stored and on the storage environment.

If the system contains N items being reworked at a constant rate ρ, then N/ρ time units will be needed to "turn over" the inventory. Thus the age of an item selected for rework under the First In, First Out (FIFO) policy will be N/ρ , and its reliability R will be

$$R = R_o e^{-\frac{Nb}{\rho}} \quad .$$

From (1) the item's reliability following the first rework will be

$$R_1 = \alpha + (1-\alpha) R_o e^{-\frac{Nb}{\rho}} \quad . \qquad (3)$$

This item will then have reliability

$$R = R_1 e^{-\frac{Nb}{\rho}}$$

immediately prior to its second rework and reliability

$$R_2 = \alpha + (1-\alpha) R_1 e^{-\frac{Nb}{\rho}}$$

following its second rework. Replacing R_1 by

$\alpha + (1-\alpha) R_0 e^{-\frac{Nb}{\rho}}$ from (3), the reliability

R_2 following the second rework becomes

$$R_2 = \alpha + \alpha(1-\alpha) e^{-\frac{Nb}{\rho}} + R_0 (1-\alpha)^2 e^{-\frac{2Nb}{\rho}} .$$

Similarly, following the third rework;

$$R_3 = \alpha + (1-\alpha) R_2 e^{-\frac{Nb}{\rho}} ,$$

or

$$R_3 = \alpha + \alpha(1-\alpha) e^{-\frac{Nb}{\rho}} + \alpha(1-\alpha)^2 e^{-\frac{2Nb}{\rho}} + R_0 (1-\alpha)^3 e^{-\frac{3Nb}{\rho}} .$$

In general then, immediately following the mth rework, the reliability R_m of an item will be

$$R_m = \sum_{i=0}^{m-1} \left[\alpha(1-\alpha)^i e^{-\frac{iNb}{\rho}} \right] + R_0 (1-\alpha)^m e^{-\frac{mNb}{\rho}} . \qquad (4)$$

for a closed system of inventory size N, constant rework rate ρ and rework effectiveness α. Values of R_m are shown in Table (I) for various values of R_0, α, and Nb/ρ.

Two interesting effects occur in these numerical examples. The first is that as the reworks progress (i.e., as m increases) the effect of each successive rework is to restore the reliability of an item to that of the previous rework. Thus in these examples a steady state is reached by the start of the fourth rework and $R_{m+1}=R_m$. The second is that the impact of the initial reliability is essentially lost by the fourth rework. These effects can be seen from expression (4) which is linear in the initial R_0.

TABLE (I)

Showing the reliability R_m following rework m with initial reliability R_0, rework effectiveness α and reliability deterioration factor Nb/ρ .

	Initial Reliability R_o =0.8					
	α =0.7 rework effectiveness			α =0.9		
Nb/ρ =	.1	.25	.4	.1	.25	.4
$R_m = R_1$	.917	.887	.861	.972	.962	.954
R_2	.949	907	873	.988	975	965
R_3	.958	.912	.876	.989	.976	.965
R_4	.960	.913	876	.990	.976	.965
	Initial Reliability R =0.9					
	α =0.7 rework effectiveness			α =0.9		
Nb/ρ =	.1	.25	.4	.1	.25	.4
$R_m = R_1$	.944	.910	.881	.981	.970	.960
R_2	.956	.913	.877	.989	.976	.964
R_3	.960	.913	.876	.989	.976	.965
R_4	.960	913	.876	.989	.976	.965
	Initial Reliability R_o =0.95					
	α =0.7 rework effectiveness			α =0.9		
Nb/ρ =	.1	.25	.4	.1	.25	.4
$R_m = R_1$	.958	.922	.891	.986	.974	.964
R_2	.960	.915	.879	.989	.976	965
R_3	.961	.914	.878	.990	.976	.965
R_4	.961	.914	.877	.990	.976	.965

As m increases (and hence i) the terms $e^{-\frac{iNb}{\rho}}$ and $e^{-\frac{mNb}{\rho}}$ become insignificantly small and add no contribution to the reliability R_m.

There are three possible conditions regarding the effect of the rework process on the reliability of an item; namely, that rework makes an item (1) better than new, (2) as good as new or, (3) not as good as new. By considering the simple case (2) "as good as new", a relationship between the initial reliability R_0 and the rework effectiveness α can be determined.

By setting the reliability following the first rework R_1 equal to the initial reliability R_0 (from (3) above)

$$R_1 = R_0 = \alpha + (1-\alpha) R_0 e^{-\frac{Nb}{\rho}}$$

the rework effectiveness α , for the case where rework restores initial reliability, becomes

$$\alpha = \frac{R_0\left(1-e^{-\frac{Nb}{\rho}}\right)}{1-R_0 e^{-\frac{Nb}{\rho}}} , \qquad (5)$$

or

$$1-\alpha = \frac{1-R_0}{1-R_0 e^{-\frac{Nb}{\rho}}} .$$

This result is depicted in Figure (2). Graphically, the interpretation is that in order to restore an item to its initial reliability, the rework effectiveness (i.e., the quality of the rework) must increase as the initial reliability increases. The rework effectiveness can be enhanced by increasing either the rework rate ρ or the rework capacity, by reducing the inventory size N or by improving the storage environment, which would reduce the reliability deterioration rate b. Additionally the following consIusions may be drawn:

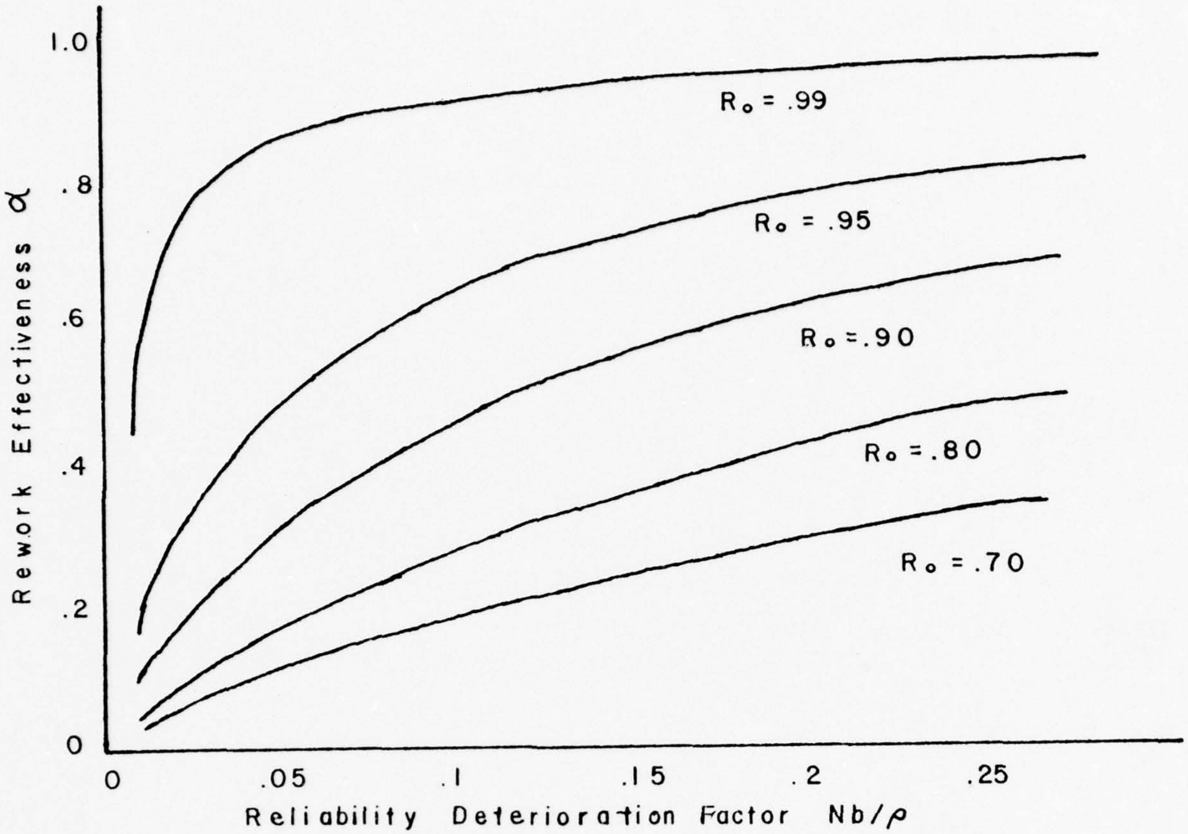

FIGURE 2. Rework Effectiveness α Necessary to Restore an Item's Reliability to "As Good as New".

for the reliability of an item following rework to be

(1) better than new $(R_{m+1} > R_m)$

$$\alpha > \frac{R_o\left(1 - e^{-\frac{Nb}{\rho}}\right)}{1 - R_o e^{-\frac{Nb}{\rho}}},$$

(2) as good as new $(R_{m+1} = R_m)$

$$\alpha = \frac{R_o\left(1 - e^{-\frac{Nb}{\rho}}\right)}{1 - R_o e^{-\frac{Nb}{\rho}}},$$

and

(3) not as good as new $(R_{m+1} < R_m)$

$$\alpha < \frac{R_o\left(1 - e^{-\frac{Nb}{\rho}}\right)}{1 - R_o e^{-\frac{Nb}{\rho}}}.$$

The transient, or "start-up" phase, where new items are entering an initially empty storage system and no rework is being done, is discussed in the next chapter. Expressions for (1) the average, or expected, reliability for the system and (2) the probability that an item's reliability are developed.

III. THE "START-UP" PHASE

An important aspect of the closed storage-rework system under consideration in this thesis is the "start-up" phase. What happens to the expected system reliability when new items are being introduced into an initially empty storage sytem? During the "start-up" phase the system receives new items into storage at an input rate ρ' and no items are being reworked. Figure (3) shows the system during the "start-up" phase.

In this chapter, expressions for the expected reliability $E[R]$ of the system and the probability that the reliability of an item will exceed a required reliability will be developed. These expressions will aid an inventory manager or high level planner in developing a replacement policy or determining the need to increase the input rate to maintain a higher level of expected reliability.

A. EXPECTED RELIABILITY

The expression for the expected reliability $E[R]$ for the items in storage during the "start-up" phase will be developed by finding the distribution of reliability $g(r)$ for the stored items and integrating it over the range of reliabilities for the items. By considering the system as static (i.e., taking a "snapshot" of the system) at time t with n new items in storage, the expected reliability expression can be developed in terms of the number of items n in storage. Since the inventory manager presumably knows how many items he has on hand, he can then readily calculate the expected reliability for the system.

Since the n items in storage arrived at a constant rate ρ', the distribution of age over the items is uniform; namely,

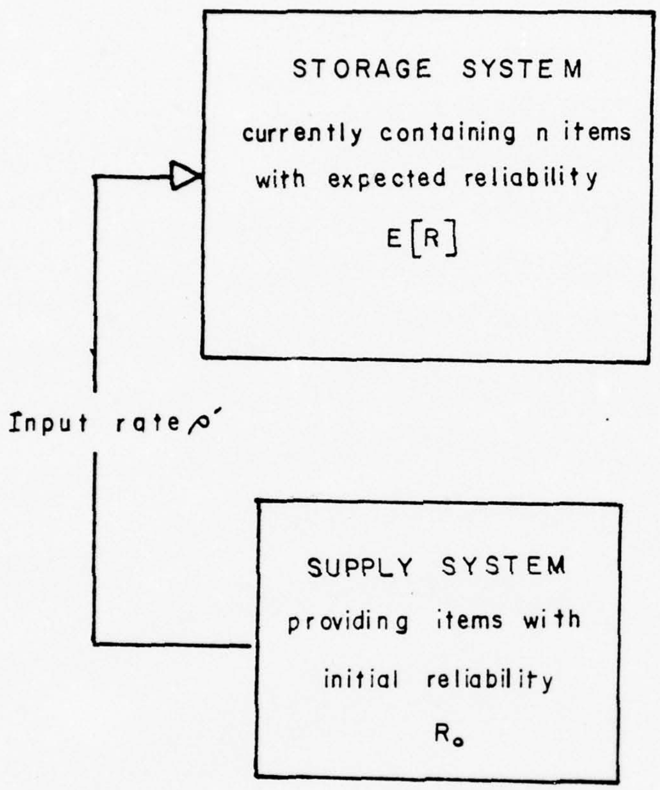

FIGURE 3. The Closed Storage System During the "start-up" Phase.

$$f(t) = \rho'/n \quad , \quad 0 \le t \le n/\rho' .$$

An item of age t which entered the storage system with an initial reliability R_0 will have reliability (from (2) above)

$$R = R_0 e^{-bt} \qquad , \quad 0 \le t \le n/\rho' ,$$

and thus

$$t = \frac{1}{b}\left(\ln R_0 - \ln R\right) \quad , \quad R_0 e^{-\frac{nb}{\rho'}} \le R \le R_0 .$$

In order to obtain the reliability density function g (R) a change of random variable is necessary, from time t to reliability R.

From [3], $\qquad g(R) = f(t) \cdot |J|$

where the Jacobian J is dt/dR, or

$$J = -1/bR$$

From this, the reliability density function g(R) can be obtained as

$$g(R) = \frac{\rho'}{nbR} \quad .$$

The expected reliability E [R] for the system can now be determined. In general, the expected value of R is [4]

$$E[R] = \int_{-\infty}^{\infty} R\, g(R)\, dR .$$

This becomes

$$E[R] = \int_{R_0 e^{-\frac{nb}{\rho'}}}^{R_0} \frac{R\rho'}{nbR}\, dR = \frac{\rho'}{nb}\left[R_0 - R_0 e^{-\frac{nb}{\rho'}}\right]$$

or

$$E[R] = \frac{R_0 \rho'}{nb}\left[1 - e^{-\frac{nb}{\rho'}}\right] . \qquad (6)$$

Notice that the expected reliability of the stored items is directly proportional to the initial reliability R_0. Also, by letting $d = nb/\rho'$ as the reliability deterioration factor, expression (6) is of the form

$$E[R] = R_0 \left[\frac{1-e^{-d}}{d} \right]$$

as shown in Figure (4). Recall that this is in essense a "snapshot" of the system during the "start-up" phase when it contains n items.

Notice also that with n very small (early in the "start-up" phase) the items in storage have not had time to deteriorate much and therefore have an expected reliability very close to the initial reliability R_0.

B. PROBABILITY THAT THE RELIABILITY OF AN ITEM EXCEEDS A MINIMUM RELIABILITY REQUIREMENT

Another useful piece of information for the inventory manager or high level planner is the probability that the reliability of an item exceeds a reliability requirement R*. If he should have a replacement decision criteria involving a minimum acceptable probability $P_r(R \geq R^*)$ that an item will exceed the reliability requirement, then such a probability statement will help him determine when replacement is required. Additionally, in the case of stored ordnance, a probability statement would enable planners to estimate the degree of success to expect from this stock of ordnance.

If R* is the reliability requirement for the stored items, then the condition to assure that all n of the stored items meet this requirement would be

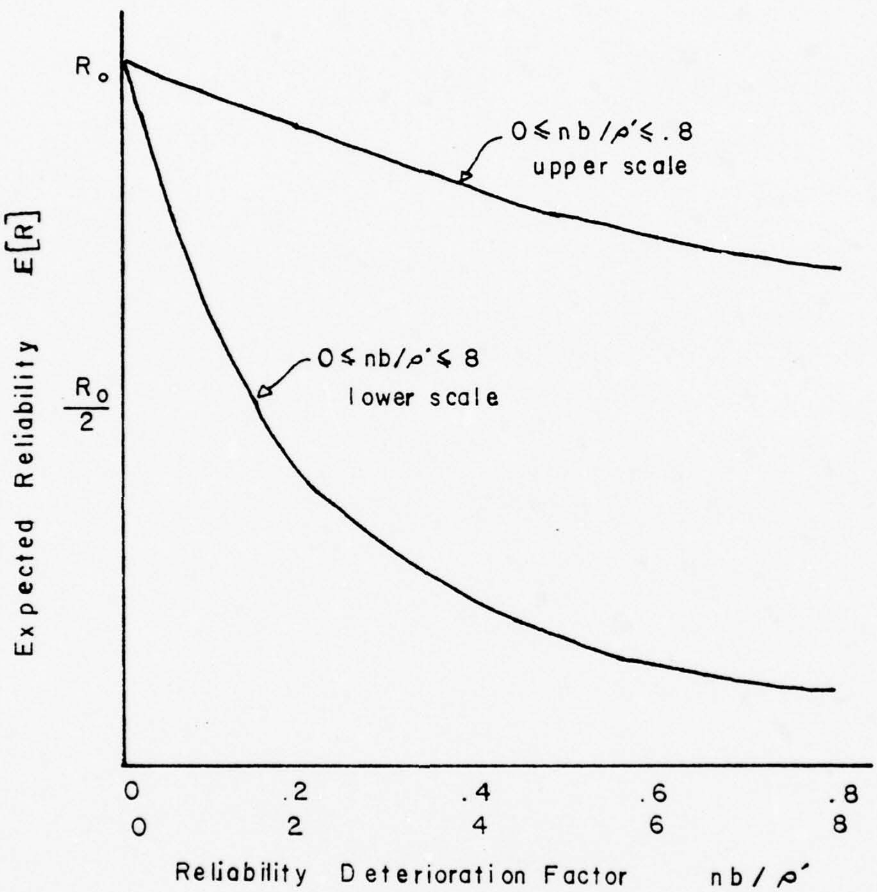

FIGURE 4. Expected Reliability of the Storage System During the "Start-Up" Phase.

$$R^* \leq R_o e^{-\frac{nb}{\rho'}} ,$$

or

$$\frac{nb}{\rho'} \leq \ln\left(\frac{R_o}{R^*}\right) .$$

Recall that n/ρ' is a measure of the time that the first, or oldest, item has been in storage. If the required reliability R* is greater than the minimum reliability of the stored items, then the proportion of the items meeting this requirement is

$$\int_{R^*}^{R_o} g(R)\,dR = \int_{R^*}^{R_o} \frac{\rho'}{nbR}\,dR ,$$

or

$$P_r(R \geq R^*) = \frac{\rho'}{nb} \ln\left[\frac{R_o}{R^*}\right] . \qquad (7)$$

This probability statement must be used with care. Since the initial reliability R_0 is usually greater than the required reliability R* and the minimum reliability is determined by the age of the oldest item in the system (i.e., n/ρ'), all the items will have reliability exceeding the requirement until the reliability of the oldest item deteriorates to the minimum required reliability. This occurs when

$$R_o e^{-\frac{nb}{\rho'}} = R^*$$

or

$$n = \frac{\rho'}{b} \ln\left[\frac{R_o}{R^*}\right] .$$

with this, the probability statement (7) becomes

$$P_r(R \geq R^*) \begin{cases} 1 & , \quad 0 \leq n \leq \frac{\rho'}{b} \ln\left[\frac{R_o}{R^*}\right] \\ \frac{\rho'}{nb} \ln\left[\frac{R_o}{R^*}\right] , & \frac{\rho'}{b} \ln\left[\frac{R_o}{R^*}\right] \leq n \leq N \,. \end{cases} \tag{8}$$

where N is the total number of items that will eventually be in the system.

Now by establishing some minimum acceptable probability Pr(R R*) that the reliability of an item will exceed a required reliability, the inventory manager can directly determine when the overall system probability will fall below the minimum acceptable. Figure (5) show the general relationship between the probability $Pr(R \geq R^*)$ and the number of items in the system for various ratios of initial reliability to required reliability (Ro/R*) during the "start-up" phase.

C. NUMERICAL EXAMPLES

For the numerical examples in the remainder of this thesis, hypothetical values for initial reliability R_0, reliability deterioration nb/ρ' and rework effectiveness α are used. A value of nb/ρ' =0.25 is used for illustrative purposes as it seems reasonable that the range of values for the reliability deterioration factor should be between 0.01 and 0.5. At the lower end of this range, the reliability of an item will deteriorate barely 1% after entering the system, hardly enough to warrant commencement of rework. At the other end, with nb/ρ' =0.5, the reliability will have decreased almost 40%, which would certainly be cause to commence rework immediately or replace the items altogether.

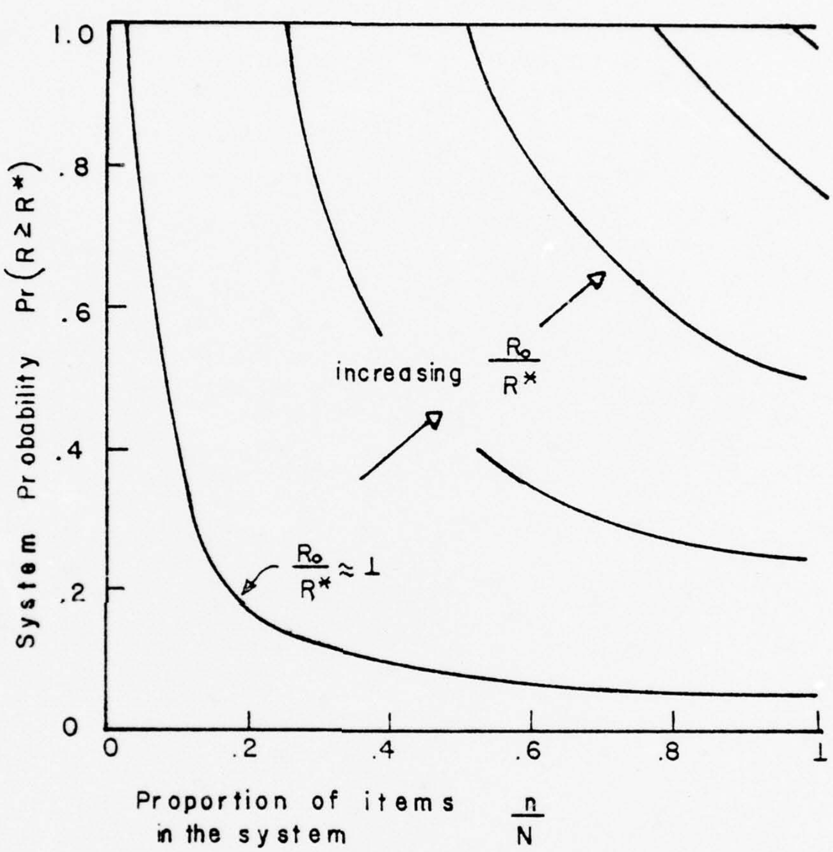

FIGURE 5. General Relationship Between $Pr(R \geq R^*)$ and the Number of Items in the System During the "Start-Up" Phase.

Using this value of nb/ρ' =0.25 and initial reliability R_0=0.95, Figure (6) shows the probability that the reliability of an item will exceed a required reliability for the closed inventory system during the "start-up" phase.

The development of equations continues in the next chapter for the expected reliability of the system and the probability of an item's reliability meeting or exceeding a required reliability for continuous rework cycles.

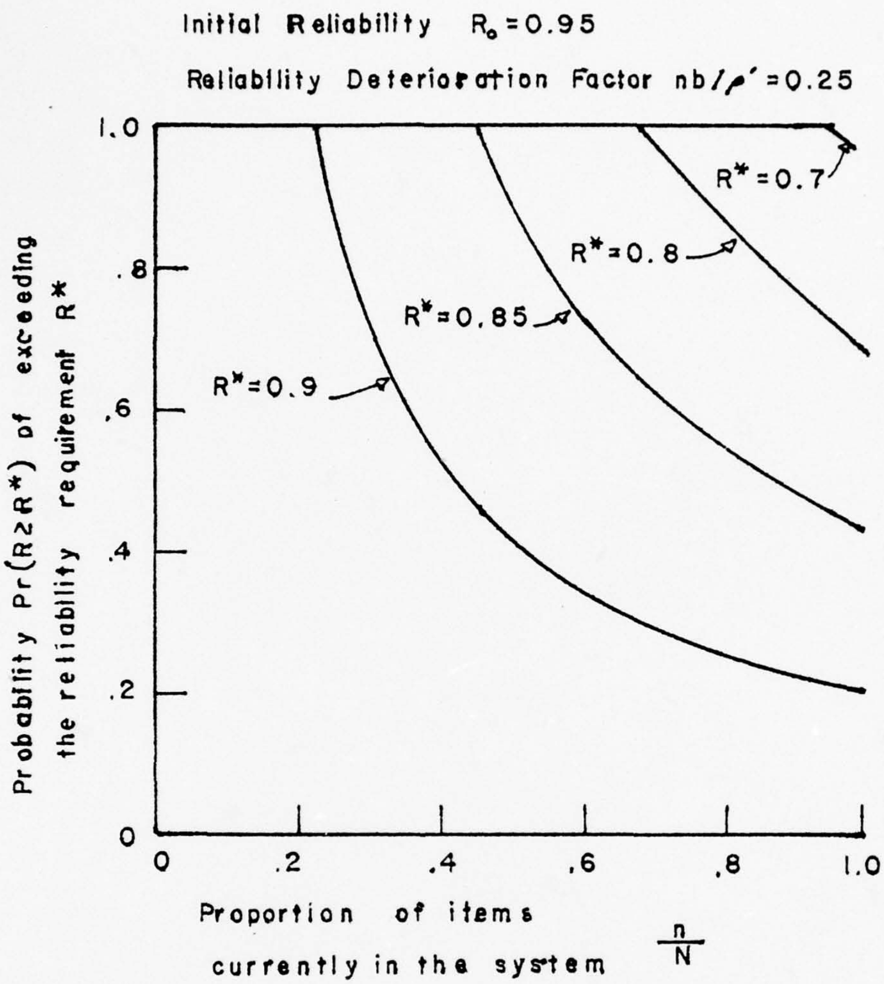

FIGURE 6. Probability Pr $(R \geq R^*)$ That the Reliability R of an Item Will Exceed a Required Reliability R* During the Start-Up" phase.

IV. CONTINUOUS REWORK

Once the storage system contains the requisite number of items N, continuous rework begins. Now, the oldest items are removed from storage, reworked, and returned to storage. This continues until the items are replaced by new or different items or expended. The items might be expended by disposing of them should they become obsolete or, in the case of ordnance, using them in a war or for training. If items are removed at random, the system model is the same, with a smaller N.

The items are reworked at a rate ρ which is assumed to be equal to the input rate ρ'. The assumption of a rework rate equal to the input rate seems reasonable if the manufacturing or supplying organization is also the rework facility as might be the case in ordnance or electronics plants. This assumption is not necessary if the reliability of the system need not be considered until after every item has been reworked at least once.

In this chapter expressions for the expected, or average reliability $E[R]$ for the system and probability statements $Pr(R \geq R^*)$ that the reliability of an item exceeds a required reliability will be developed.

A. EXPECTED RELIABILITY

One of the important things an inventory manager or combat planner might want to know is the expected reliability $E[R]$ of all the items in storage. Such information would help him to decide when to begin replacing the stored item, or upgrade the quality of the rework when the expected reliability falls below a selected level.

In developing the expressions for expected reliability it is necessary to keep in mind two distinct sets of items in the storage system: (1) those items which have undergone rework m+1 and (2) those awaiting the m+1 st rework. With a total number of items in storage N, there are n items which have been reworked m+1 times and N-n items which have been reworked only m times.

The n items which have been reworked m+1 times are considered first. The development of the expected reliability $E_1[R]$ expression for this set of items is identical to that of the "start-up" phase in the previous chapter. The age density function $f_1(t)$ is uniform:

$$f_1(t) = \rho/n \quad , \qquad 0 \le t \le n/\rho \; .$$

An item of age t which entered the storage system with reliability R_{m+1} after the m+1 st rework will have reliability

$$R(t) = R_{m+1}\, e^{-bt} \quad , \qquad 0 \le t \le n/\rho \; ,$$

and thus

$$t = \frac{1}{b}\left(\ln R_{m+1} - \ln R\right) \; , \quad R_{m+1}\, e^{-\frac{nb}{\rho}} \le R \le R_{m+1} \; .$$

The Jacobian is still

$$J = dt/dR = -1/bR \; ,$$

and the reliability density function is

$$g_1(R) = \frac{\rho}{nbR}$$

The expected reliability for the n items which have been reworked m+1 times is

$$E_1[R] = R_{m+1}\frac{\rho}{nb}\left[1 - e^{-\frac{nb}{\rho}}\right] \; . \qquad (9)$$

The remaining N-n items which have been reworked m times are now considered. The distribution of age over these items is also uniform, but with an age density function of

$$f_2(t) = \frac{\rho}{N-n} \quad , \qquad n/\rho \le t \le N/\rho \ .$$

Again, an item of age t will have reliability

$$R = R_m e^{-bt} \quad , \qquad n/\rho \le t \le N/\rho \ ,$$

or

$$t = \frac{1}{b}\left(\ln R_m - \ln R\right) , \quad R_m e^{-\frac{Nb}{\rho}} \le R \le R_m e^{-\frac{nb}{\rho}} \ .$$

The Jacobian is still

$$J = -1/bR$$

and the reliability density function is

$$g_2(R) = \frac{\rho}{(N-n)bR} \ .$$

The expected reliability for the items awaiting the m+1 st rework is

$$E_2\left[R\right] = \int_{R_m e^{-\frac{Nb}{\rho}}}^{R_m e^{-\frac{nb}{\rho}}} \frac{R\rho}{(N-n)bR} \, dR \quad ,$$

or

$$E_2\left[R\right] = \frac{R_m \rho}{(N-n)b}\left[e^{-\frac{nb}{\rho}} - e^{-\frac{Nb}{\rho}}\right] \ . \qquad (10)$$

The expected reliability $E\left[R\right]$ for the entire system is the sum of the expectations of the two sets of items multiplied by the fraction of the system each represents; namely,

$$E[R] = \frac{n}{N} E_1[R] + \frac{(N-n)}{N} E_2[R] \, ,$$

or

$$E[R] = R_{m+1} \frac{\rho}{Nb}\left[1 - e^{-\frac{nb}{\rho}}\right] + R_m \frac{\rho}{Nb}\left[e^{-\frac{nb}{\rho}} - e^{-\frac{Nb}{\rho}}\right] \, . \quad (11)$$

Notice that the average reliability of all the stored items is directly proportional to the reliability following rework (R_m and R_{m+1}) and therefore proportional to the effectiveness of the rework α and the initial input *reliability Ro. Two other observations may be made.* First, that if successive rework essentially restores the reliability of the previous rework (i.e., $R_{m+1} = R_m$) as was suggested by Table (I) beyond the fourth rework, then the average reliability becomes

$$E[R] = \frac{R_m \rho}{Nb}\left[1 - e^{-\frac{Nb}{\rho}}\right] .$$

Second, when the M+1 st rework has just started, implying n very small, the average system reliability will be very close to that of the items which have been reworked m times:

$$\lim_{n \to 0} E[R] = \frac{R_m \rho}{Nb}\left[1 - e^{-\frac{Nb}{\rho}}\right] .$$

Similarly, when all the items have completed the m+1 st rework, the average system reliability becomes

$$\lim_{n \to N} E[R] = \frac{R_{m+1} \rho}{Nb}\left[1 - e^{-\frac{Nb}{\rho}}\right] .$$

Now, by letting d = Nb/ρ as a factor proportional to the "turnover" time (or rework interval) and a= n/N, the average system reliability

expression (11) can be conveniently written as

$$E[R] = R_{m+1}\left[\frac{1-e^{-ad}}{d}\right] + R_m\left[\frac{e^{-ad}-e^{-d}}{d}\right] .$$

This expression will be graphically displayed in Figure (7) and discussed in the Numerical Example section at the end of this chapter.

B. PROBABILITY THAT THE RELIABILITY OF AN ITEM EXCEEDS A MINIMUM RELIABILITY REQUIREMENT.

As mentioned in the previous chapter, the probability that an item's reliability exceeds some required reliability R* is useful information. In addition to helping establish a replacement criteria for the items in storage, a probability statement might be used to estimate the expected degree of success from a stock of ordnance.

The development of the system probability statement is similar to that of the expected reliability expression, that is, by considering the two subsystems (the n items reworked m+1 times and the N-n reworked m times) and then conbining the results.

There are several situations which must be considered. One is where the minimum reliability of an item in the system $R_m e^{-\frac{Nb}{\rho}}$ (for rework m) exceeds the required reliability R*. That is

$$R_m e^{-\frac{Nb}{\rho}} \geq R^* ,$$

or

$$\frac{Nb}{\rho} \leq \ln\left[\frac{R_m}{R^*}\right] .$$

In this situation all the items in the system exceed the reliability requirement and

$$P_r(R \geq R^*) = 1 \; .$$

A second situation is that the items reworked m times always exceed the reliability requirement, and the items reworked m+1 times initially exceed the requirement, but later fall below it, i.e., $R_{m+1} > R^*$ and $R_{m+1} e^{-\frac{Nb}{\rho}} < R^*$. In this situation the system will have $P_r(R \geq R^*) = 1$ from the commencement of the M+1 st rework until the time that the first item reworked to R_{m+1} deteriorates to less than the required reliability. This occurs when

$$R_{m+1} e^{-\frac{nb}{\rho}} = R^*$$

or

$$n = \frac{\rho}{b} \ln \left[\frac{R_{m+1}}{R^*} \right] \; .$$

At this time the system probability $Pr(R \geq R^*)$ falls below 1.

The proportion of the items meeting the reliability requirement is

$$\int_{R^*}^{R_{m+1}} g(R)\, dR = \int_{R^*}^{R_{m+1}} \frac{\rho}{nbR}\, dR = \; , \quad n \geq \frac{\rho}{b} \ln \left[\frac{R_{m+1}}{R^*} \right] ,$$

thus

$$P_r(R \geq R^*) = \frac{\rho}{nb} \ln \left[\frac{R_{m+1}}{R^*} \right] , \quad n \geq \frac{\rho}{b} \ln \left[\frac{R_{m+1}}{R^*} \right] . \quad (12)$$

Recall that expression (12) is true only for the N items reworked m+1 times. The probability statement for the entire system can then be written as

$$P_r(R \geq R^*) = \begin{cases} 1 & , \quad 0 \leq n \leq \frac{\rho}{b} \ln\left[\frac{R_{m+1}}{R^*}\right] \\ \left(1 - \frac{n}{N}\right) + \frac{\rho}{Nb} \ln\left[\frac{R_{m+1}}{R^*}\right] & , \quad \frac{\rho}{b} \ln\left[\frac{R_{m+1}}{R^*}\right] \leq n \leq N \end{cases} \qquad (13)$$

for the boundary conditions:

(1) $R_m > R_{m+1} > R^*$ and

(2) $R_m e^{-\frac{Nb}{\rho}} \geq R^* \geq R_{m+1} e^{-\frac{Nb}{\rho}}$

This expression (13) states that all the items have reliability exceeding the required reliability until the number of items n reworked m+1 times exceed an amount $(\rho/b) \ln (R_{m+1}/R^*)$. This corresponds to the time when some of the items have reliability which is falling below the required reliability. At this point, the probability becomes linear in n with the term (1-n/N) representing the diminishing contribution of those items having finished only the mth rework, all of which have reliability exceeding the requirement. The second term $(\rho/Nb) \ln (R_{m+1}/R^*)$ represents the items reworked m+1 times. This term resembles expression (12), but recall that $n < N$ which means $(\rho/Nb < \rho/nb)$ and the contribution of these items (reworked m+1 times) increases in significance as the m+1 st rework progresses (i.e., as n approaches N) and the influence of the mth rework decreases.

The third situation which needs to be considered is the reverse of the one above. That is, the items reworked m+1 times exceed the reliability required and those reworked m times do not. This might be the case when each successive rework restores the reliability of an item to level higher than the previous rework, or makes the item "better than new". In this situation $R_{m+1} > R_m$ and the items reworked m+1 times will have Pr $(R \geq R^*) = 1$.

The N-n items reworked m times now need to be considered carefully. Recall that once the m+1 rework begins, rework m ends, and the items being returned to storage have reliability R_{m+1}. The N-n items having input reliability R_m now have a maximum reliability $R_m e^{-\frac{nb}{\rho}}$ which is constantly decreasing as rework m+1 progresses. Early in the m+1 st rework, many of the N-n items reworked m times will have reliability exceeding the requirement. Once the maximum reliability for these items $R_m e^{-\frac{nb}{\rho}}$ falls below the reliability requirement R*, the entire set will have reliability less than the requirement and then Pr $(R \geq R^*)=0$. Until this occurs, at $n \geq (\rho/b)\ln(R_m/R^*)$, the proportion of items exceeding the requirement is

$$\int_{R_{min}}^{R_{max}} g_2(R)\,dR = \int_{R^*}^{R_m e^{-\frac{nb}{\rho}}} \frac{\rho}{(N-n)bR}\,dR \quad , \quad 0 \leq n \leq \frac{\rho}{b}\ln\left[\frac{R_m}{R^*}\right],$$

or

$$P_r(R \geq R^*) = \frac{\rho}{(N-n)b}\left[\ln\left(\frac{R_m}{R^*}\right) - \frac{nb}{\rho}\right], \quad 0 \leq n \leq \frac{\rho}{b}\ln\left[\frac{R_m}{R^*}\right] \quad (14)$$

The probability statement for the system can now be written as

$$P_r(R \geq R^*) = \begin{cases} \frac{\rho}{Nb}\ln\left[\frac{R_m}{R^*}\right] & , \quad 0 \leq n \leq \frac{\rho}{b}\ln\left[\frac{R_m}{R^*}\right] \\ \frac{n}{N} & , \quad \frac{\rho}{b}\ln\left[\frac{R_m}{R^*}\right] \leq n \leq N \end{cases} \quad (15)$$

for the boundary conditions

(1) $R_{m+1} > R_m > R^*$ and

(2) $R_{m+1}e^{-\frac{Nb}{\rho}} \geq R^* \geq R_m e^{-\frac{Nb}{\rho}}$.

Notice that the probability statement (15) shows the dominant effect of the mth rework until such time as none of the items reworked m times have reliability exceeding the requirement. Then the system probability becomes linear in n tending toward 1 as n approaches N.

It is possible that the system probability $Pr(R \geq R^*)$ does not fall to an unacceptably low level under the conditions of expression (11). What must be done now is to investigate the fourth situation, where the minimum reliabilities resulting from both the mth and the M+1 st rework fall below the required reliability. That is,

(1) $R_m e^{-\frac{Nb}{\rho}} < R^*$, and

(2) $R_{m+1} e^{-\frac{Nb}{\rho}} < R^*$.

In this situation there are four subsets of items which need to be considered:

1. Items reworked m+1 times, whose minimum reliability exceeds the required reliability. This occurs during the early portion of the m+1 st rework when $R_{m+1} e^{-\frac{nb}{\rho}} \geq R^*$, or $n \leq (\rho/b) \ln(R_{m+1}/R^*)$. For these n items

$$P_r(R \geq R^*) = 1 \quad , \qquad 0 \leq n \leq (\rho/b) \ln(R_{m+1}/R^*) .$$

2. Items reworked m+1 times, whose minimum reliability is less than that required. This has already been described above as expression (12) or

$$P_r(R \geq R^*) = \frac{\rho}{nb} \ln\left[\frac{R_{m+1}}{R^*}\right] \quad , \qquad \frac{\rho}{b} \ln\left[\frac{R_{m+1}}{R^*}\right] \leq n \leq N .$$

3. Items reworked m times whose maximum reliability exceeds the required reliability. This situation is identical to the case described by expression (14) and can be expressed as

$$P_r(R \geq R^*) = \frac{\rho}{(N-n)b}\left[\ln\left(\frac{R_m}{R^*}\right) - \frac{nb}{\rho}\right] , \qquad 0 \leq n \leq \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] .$$

4. Items reworked m times whose maximum reliability is less than the required reliability, i.e., $R_m e^{-\frac{nb}{\rho}} < R^*$ or $n > (\rho/b) \ln(R_m/R^*)$. This occurs toward the end of the m+1 st rework and

$$P_r(R \geq R^*) = 0 \quad , \qquad \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] \leq n \leq N .$$

Now, before these last four separate probability statements can be combined into one system probability statement, some relationship between R_m and R_{m+1} must be defined. As mentioned in Chapter II, there are three conditions which may exist regarding the effectiveness of the

rework mechanism:

(1) $R_{m+1} > R_m$, where the rework mechanism makes an item "better than new",

(2) $R_{m+1} = R_m$, where the rework mechanism makes an item "as good as new", and

(3) $R_{m+1} < R_m$, where the rework mechanism does not make an item as "good as new".

Considering each condition in turn:

(1) $R_{m+1} > R_m$:

$$P_r(R \geq R^*) = \begin{cases} \dfrac{\rho}{Nb} \ln\left[\dfrac{R_m}{R^*}\right], & 0 \leq n \leq \dfrac{\rho}{b}\ln\left[\dfrac{R_m}{R^*}\right] \\ \dfrac{n}{N}, & \dfrac{\rho}{b}\ln\left[\dfrac{R_m}{R^*}\right] \leq n \leq \dfrac{\rho}{b}\ln\left[\dfrac{R_{m+1}}{R^*}\right] \\ \dfrac{\rho}{Nb} \ln\left[\dfrac{R_{m+1}}{R^*}\right], & \dfrac{\rho}{b}\ln\left[\dfrac{R_{m+1}}{R^*}\right] \leq n \leq N . \end{cases} \tag{16}$$

(2) $R_{m+1} = R_m$:

$$P_r(R \geq R^*) = \frac{\rho}{Nb} \ln\left[\frac{R_m}{R^*}\right], \quad 0 \leq n \leq N . \tag{17}$$

(3) $R_{m+1} < R_m$:

$$P_r(R \geq R^*) = \begin{cases} \dfrac{\rho}{Nb} \ln\left[\dfrac{R_m}{R^*}\right], & 0 \leq n \leq \dfrac{\rho}{b}\ln\left[\dfrac{R_{m+1}}{R^*}\right] \\ \dfrac{\rho}{Nb} \ln\left[\dfrac{R_m}{R^*} \cdot \dfrac{R_{m+1}}{R^*}\right] - \dfrac{n}{N}, & \dfrac{\rho}{b}\ln\left[\dfrac{R_{m+1}}{R^*}\right] \leq n \leq \dfrac{\rho}{b}\ln\left[\dfrac{R_m}{R^*}\right] \\ \dfrac{\rho}{Nb} \ln\left[\dfrac{R_{m+1}}{R^*}\right], & \dfrac{\rho}{b}\ln\left[\dfrac{R_m}{R^*}\right] \leq n \leq N . \end{cases} \tag{18}$$

These three probability statements (16, 17 & 18) all have the boundary conditions:

(1) $R_m > R^*$,

(2) $R_{m+1} > R^*$,

(3) $R_m e^{-\frac{Nb}{\rho}} < R^*$, and

(4) $R_{m+1} e^{-\frac{Nb}{\rho}} < R^*$.

C. NUMERICAL EXAMPLES

Continuing the example of Chapter III, using initial reliability $R_0 = .95$, reliability deterioration factor $Nb/\rho = .25$ and rework effectiveness $\alpha = .7$, Table (I) shows the reliability R_m of an item following rework m as $R_1 = .922$, $R_2 = .915$, $R_3 = .914$ and $R_4 = .914$. Figure (7) shows the expected reliability of such a system, commencing at the "start-up" phase, and continuing into the third rework cycle.

As can be seen from Figure (7), the expected reliability $E[R]$ decreases fairly rapidly during the "start-up" phase and levels off as the rework commences. By the middle of the second rework cycle, this system is essentially at the steady state condition.

Figure (8) shows the probability $Pr(R \geq R^*)$ that an item will have reliability R exceeding a required reliability R^* for the closed system from the moment new items of initial reliability R_0 begin entering the system (the "start-up" phase) and continuing through the third rework cycle. It is readily apparent that by the third rework cycle, the probability of an item of reliability R exceeding the required R^* has become almost constant, and that steady state condtions have been reached.

In the next chapter, the replacement phase is examined. During this phase, the old items are being replaced in the storage system by new or

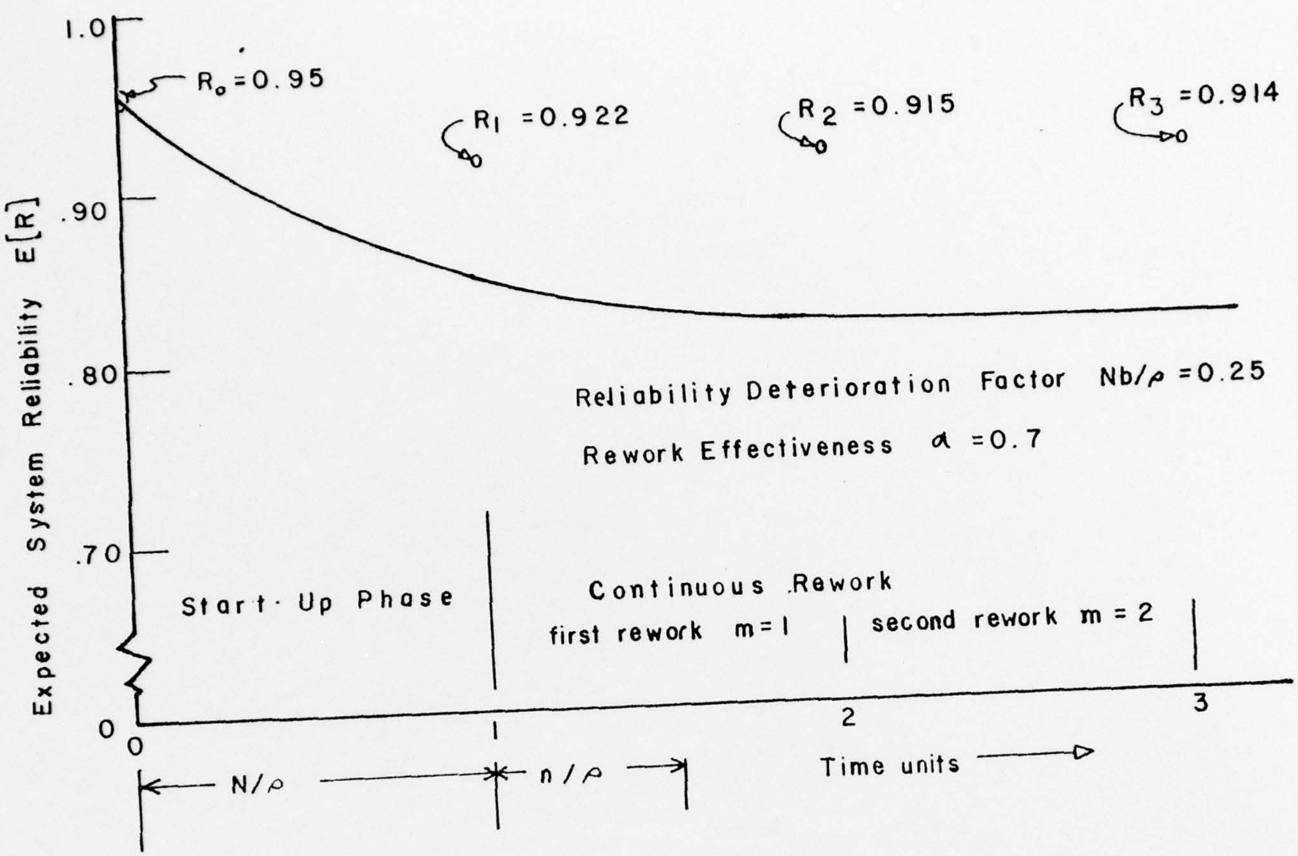

FIGURE 7. Expected Reliability For the System of Stored Items.

or different items, and no rework is being done. Expected reliability expressions and probability statements, similar to those already developed, are presented.

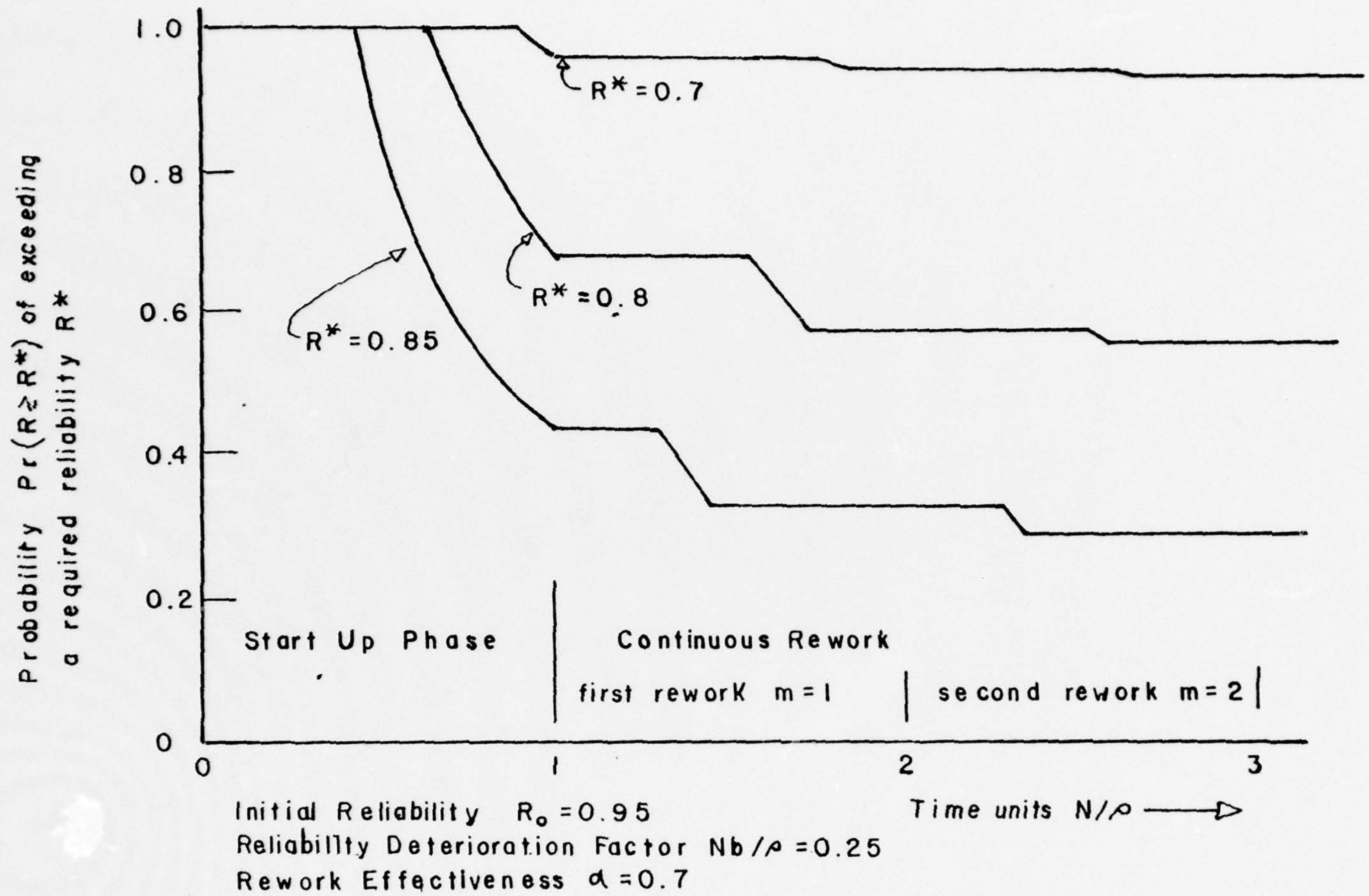

FIGURE 8. Probability Pr (R ≥ R*) That an Item Will Have Reliability R Exceeding a Required Reliability R*.

V. THE REPLACEMENT PHASE

The decision to replace the items in storage with new or improved items can be made for a variety of reasons. The old items may have deteriorated so that they cannot meet the reliability requirement, or that they are simply beyond economical repair. The new items may offer greatly increased initial reliability or expanded capability ("more bang for the buck"). Or a new rework mechanism may be developed which has a greater rework effectiveness.

This replacement phase may involve two different situations. The first is simply to retain the old items and send them through an improved rework process. This is similar to the system shown in Figure (1), with the items being reworked to initial reliability R_0'. The second situation is replacing the old items with new, as shown in Figure (9). In this situation, no rework is being performed until the system contains only new items, when the continuous rework process recommences. This rework process might also have been improved, in which case the effects of the improvement will be evident in the system after the new items are reworked.

In this chapter expressions are developed for the expected reliability of the system and the probability that an item will have reliability exceeding a required reliability, during the replacement phase.

For simplicity, the replacement phase is assumed to start after the third or fourth rework cycle commences. This implies that steady state conditions exist (i.e., $R_{m+1} = Rm$) and the shift from rework to replacement may be made during the rework cycle. The development of the expressions will also be appropriate for replacement beginning at the end of a

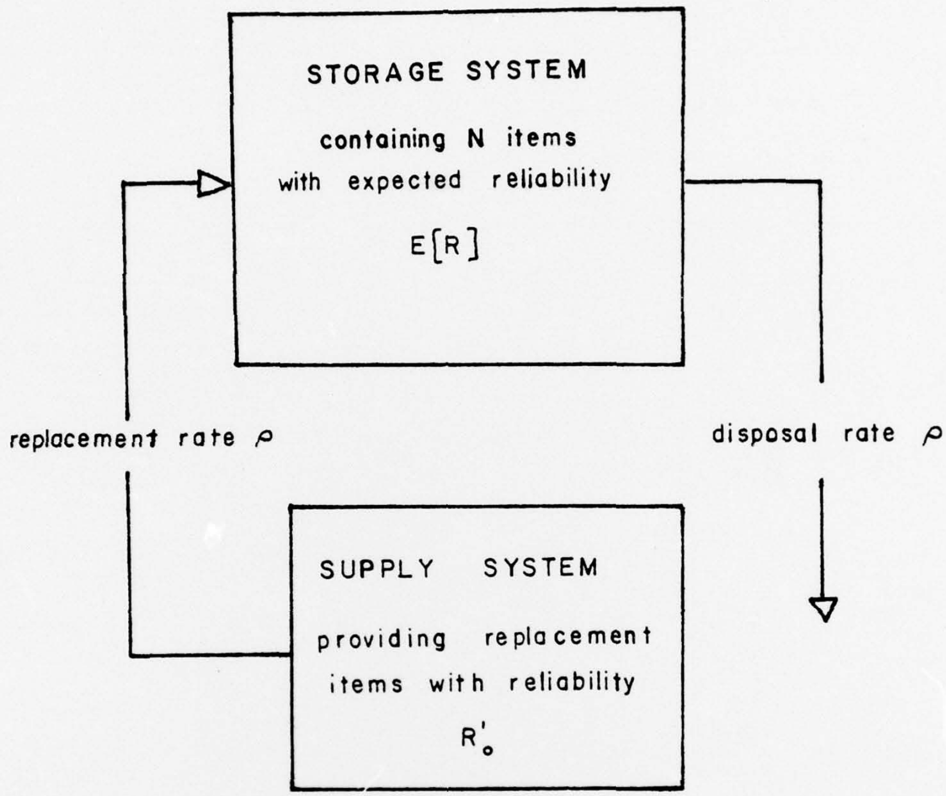

FIGURE 9. The Storage System During the Replacement Phase.

rework cycle. The effect here being that instead of items with reliability R_{m+1} entering the system, items with reliability R_0' will enter the system.

A. EXPECTED RELIABILITY

Since the storage system "sees" only two sets of items, those that had input reliability R_m (the old items) and those entering with reliability R_0' (the replacement items), the system is essentially identical to that during continuous rework. The expected reliability expression is developed in the same manner as in Chapter IV, and can be written directly from expression (11) as

$$E[R] = R_0' \frac{\rho}{Nb}\left[1-e^{-\frac{nb}{\rho}}\right] + R_m \frac{\rho}{Nb}\left[e^{-\frac{nb}{\rho}} - e^{-\frac{Nb}{\rho}}\right] \qquad (19)$$

where n is the number of items in the system with input reliability R_0.

Once all the old items are replaced in the system, the continuous rework process begins again. It is at this point that the effect of an increased rework effectiveness α will become apparent. Since no rework is being performed during the replacement phase (when new items are replacing old items), the increased effectiveness will be evident after the items go through the rework process.

B. PROBABILITY THAT THE RELIABILITY OF AN ITEM WILL EXCEEDS A MINIMUM REQUIRED RELIABILITY

The probability statements for the replacement phase are developed in a manner identical that in Chapter IV. It is possible that the input reliability R_0' of the new items is less than the reliability R_m of the old items immediately after the mth rework. This might be the case

where a new type of ordnance is being acquired at lower initial cost, but has lower initial reliability. In general, though, it is reasonable to assume the replacement items have a higher input reliability than the items they are replacing.

This is the situation which will be discussed (i.e., $R_0' > R_m$). Here, only two cases need to be considered:

(1) where the minimum reliability of the new items $R_0' e^{-\frac{Nb}{\rho}}$ is always greater than the reliability required R*, and

(2) where the minimum reliability of the new items falls below the required reliability.

The first case was developed in Chapter IV and presented as expression (15), which for this case becomes

$$Pr(R \geq R^*) = \begin{cases} \frac{\rho}{Nb} \ln\left[\frac{R_m}{R^*}\right], & 0 \leq n \leq \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] \\ \frac{n}{N}, & \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] \leq n \leq N, \end{cases} \qquad (20)$$

for the boundary conditions

(1) $R_0' > R_m > R^*$ and

(2) $R_0' e^{-\frac{Nb}{\rho}} \geq R^* \geq R_m e^{-\frac{Nb}{\rho}}$.

The second case was also developed earlier and presented as expression (16), and it becomes

$$
P_r(R \geq R^*) = \begin{cases} \frac{\rho}{Nb} \ln\left[\frac{R_m}{R^*}\right], & 0 \leq n \leq \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] \\ \frac{n}{N}, & \frac{\rho}{b} \ln\left[\frac{R_m}{R^*}\right] \leq n \leq \frac{\rho}{b} \ln\left[\frac{R_o'}{R^*}\right] \\ \frac{\rho}{Nb} \ln\left[\frac{R_o'}{R^*}\right], & \frac{\rho}{b} \ln\left[\frac{R_o'}{R^*}\right] \leq n \leq N, \end{cases} \tag{21}
$$

for the conditions:

(1) $R_o' > R_m > R^*$ and

(2) $R_m e^{-\frac{Nb}{\rho}} < R_o' e^{-\frac{Nb}{\rho}} < R^*$.

The development of the probability statements for situations other than the one used in this discussion is relatively straight-forward. If the assumption of replacing the items during steady state continuous rework (or at the end of a rework cycle) is not made, then it is necessary to keep track of three sets of items: those items reworked m and m+1 times and the replacement items. Also, if the assumption of $R_o' > R_m$ is not made, then the development of expression (18) becomes appropriate for the replacement phase.

VI. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

A simple, closed inventory storage system from which the stored items are removed, sent through a rework mechanism to improve the item's reliability, and returned to storage has been investigated in this thesis. Such a system might be a large stock of ordnance which is acquired, stored and reworked at regular intervals.

A general rework model was developed where the effect of the rework is to improve an item's reliability by an amount proportional to the effectiveness of the rework and the item's reliability just prior to rework. This model allows for a rework mechanism which may make an item "better than new", as "good as new" or not as "good as new. The numerical examples used suggested that beyond the fourth rework each successive rework restored the reliability of an item to the same level as the previous rework, in effect achieving a steady-state condition.

Expressions for the expected or average reliability of the system were developed for the entire life of the storage-rework system from the moment new items enter an initially empty system through multiple rework cycles and concluding with the replacement of the old, reworked items with new items, whereupon the multiple rework cycles would begin again. The probability that an item chosen at random might have reliability exceeding some predetermined reliability requirement was also evaluated.

There are several interesting areas which might be pursued in further study. One is to determine methods for measuring the rate of reliability deterioration b so that the expected reliability expressions

and the system probability statements developed in this thesis might be useful to an inventory manager or high level planner. Another area is to investigate the behavior of the costs involved in the storage-rework system.

There are certain costs associated with the acquisition of new items, the rate of rework and the effectiveness of the rework. One of the topics which might be addressed is the cost effectiveness of increasing the rate of rework versus increasing the effectiveness of the work to increase the expected system reliability.

Another area which could be investigated is the effect of an input to and output from the system. Taking the example of stored ordnance, a determination of the effect of ordnance expenditure, say for training purposes, on the system reliability could be both interesting and informative. Other areas of interest might be to compare a LIFO policy for expenditure vice FIFO and the impact of each on system reliability.

A reliability model has been developed in this thesis for a system of stored items requiring rework. It is hoped that the results presented here will not only be useful to inventory managers and high-level planners but will also generate further interest and study in this area.

BIBLIOGRAPHY

1. Lindsay, G.F., Reliability in Stored Ordnance: Selection of Items for Training Expenditure, paper presented to the Chief of Naval Operations (OP-964), Washington, D.D., 20 June 1974.

2. Carson, G.B., Bolz, H.A., Young, H.H., Production Handbook, The Roland Press, 1972.

3. Hogg, R.V., and Craig, A.T., Introduction to Mathematical Statistics, The Macmillan Company, 1959.

4. Feller, W., An Introduction to Probability Theory and Its Applications, John Wiley & Sons, 1957.

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